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ANALYSIS OF THE IRREGULARLY SAMPLED SIGNALS ABOVE THE NYQUIST LIMIT

A new method of spectrum analysis of the nonuniformly sampled data above the Nyquist limit is proposed, which is based on the digital spectral shift to the baseband, optional filtering and the subsequent spectrum analysis below the Nyquist limit. The method is distinguished by low aliasing frequency estimation and sufficient computation effectiveness. The mathematical background is shown and simulation results are presented to illustrate the method basics.

Keywords: Nonuniformly sampled data, digital frequency shift

1. INTRODUCTION

Nonuniformly sampled data occurs in several applications such as geophysics [1], Laser Doppler Anemometry (LDA) [2], oscilloscopes [3] and radar or sonar signal processing [4]-[5]. Such type of data is used by system designers to avoid aliasing in the signal spectrum or when, due to technical problems, it is sometimes impossible to perform regular sampling. Several methods for spectrum analysis of the nonuniformly sampled data are proposed, such as Lomb periodogram [6], Koh-Wicks-Sarkar equation [5], Dirichlet transformation [7], SECOEX method [8], non-uniform DFT [9] or some approximation methods [10]-[11]. The main disadvantages of the enumerated methods consist in the aliasing effects due to the non–orthogonal basis or their impossibility to analyze the nonuniformly sampled data above the Nyquist limit.

Therefore, this paper proposed a method to overcome the selected problems and allowed spectrum analysis above Nyquist limit. This method is based on the digital spectral shift to the baseband, optional filtering and implementation of the spectrum analysis below the Nyquist limit.

2. SPECTRUM SHIFT EQUATION DEFINITION

Spectral shift analytical expression of the nonuniformly sampled data is implemented using the basic spectral shift equation [12]:

$$U\left(e^{j2\pi\omega}\right) = X\left(e^{j2\pi(\omega-\gamma)}\right), \tag{1}$$

where: $U(j\omega)$ - shifted spectral response, γ - frequency shift.

The nonuniformly sampled data $u(t_n)$ are calculated by the well-known inverse Fourier transform [13]:

$$u(t_n) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} U(j\omega) e^{-j\omega t_n} d\omega . \qquad (2)$$

If Eq. (2) is substituted to the definition Eq. (1), the nonuniformly sampled data $u(t_n)$ are estimated according to the equation:

$$u(t_n) = x(t_n) e^{-j\gamma t_n} . \qquad (3)$$

The mathematical proof of this equation can be expressed as:

$$u(t_n) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} X(j\Omega) e^{-j(\gamma+\Omega)t_n} d\Omega = \frac{e^{-j\gamma t_n}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} X(j\Omega) e^{-j\Omega t_n} d\Omega = x(t_n) e^{-j\gamma t_n} .$$

3. METHOD DESCRIPTION

The simplest case appears when the signal frequency band is smaller than the average sampling frequency *fs* (Fig. 1). This situation occurs in the radio propagation channels where signals are transmitted in selected frequency bands (GSM, WLAN, etc.). In this case the original signal is filtered by an channel analog filter and is frequency shifted to the analyzed frequency band by using the Eq. (3).

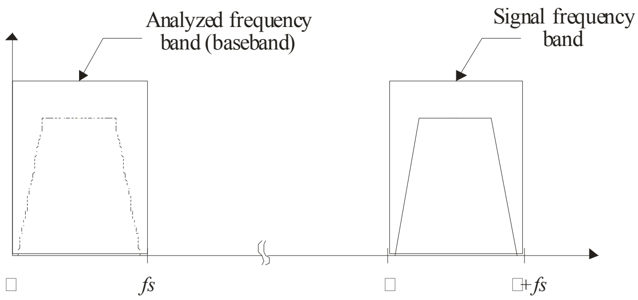


Fig. 1. Narrow signal frequency band case.

The new time series $u(t_n)$ is analyzed in the frequency domain according to equation [14]:

$$u(t_k) = \sum_{i=0}^{N-1} A_i e^{j\omega_i t_k} . \qquad (4)$$

Since the analyzed frequencies are uniformly disposed ($\omega_i = \omega_0 + i\Delta\omega, 0 \le i \le N-1$), the frequency estimation Eq. (4) is modified as:

$$u(t_k) = e^{j\omega_0 t_k} \sum_{i=0}^{N-1} A_i e^{ji\Delta\omega t_k} . \qquad (5)$$

The complex amplitude coefficients A_i are calculated using the equation:

$$A = U.Y^{-1}, \tag{6}$$

where: $A = [A_0, ..., A_{N-1}]$ - amplitude coefficient matrix, $U = [u(t_0), ..., u(t_{N-1})]$ - data matrix, $Y = \begin{pmatrix} 1 & ... & ... & 1 \\ e^{j\Delta\omega t_0} & ... & ... & e^{j\Delta\omega t_{N-1}} \\ ... & ... & ... & ... \\ e^{j(N-1)\Delta\omega t_0} & ... & ... & e^{j(N-1)\Delta\omega t_{N-1}} \end{pmatrix}$ - basic matrix.

A more complicated case appears when the signal frequency band is wider than the average sampling frequency (Fig. 2). This situation is observed in location devices (radiolocation, sonar and lidar devices, etc.). In this case the whole signal frequency band is divided into M secondary bands whose bandwidth is equal to fs. As the interpolation spectrum analysis is used to estimate the signal frequency response, the signal has to be filtered by a band-pass filter (*BPF*) for each secondary band. Therefore, the spectrum estimation procedure has to be implemented M times to calculate the entire signal frequency response.

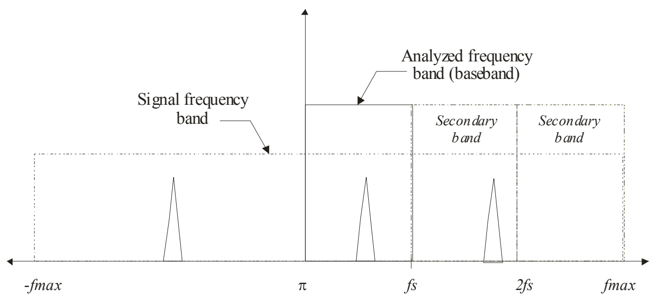


Fig. 2. Wideband case.

As the basic matrix is recognized as a Wandermonde matrix, the total number of floating point calculations is defined as $MN\log_2^2 N$ since the single-band calculations require $N\log_2^2 N$ [15-16]. If the SECOEX method is used to calculate the frequency response, the required number of calculations is equal to N^2 when the pseudo-inverse matrix is preliminarily calculated. Therefore, the SECOEX method requires approximately the same amount of calculation time, but it produces very high aliasing peaks as wideband noise [8]. The other frequency estimation methods, described in § 1, are distinguished with the same disadvantages.

4. SIMULATION RESULTS

To demonstrate the significance of the theoretical formulation, we test the proposed method using the MATLAB® routine. The simulated signal is defined as a linear frequency modulated (LFM) pulse, because its spectrum is nearly rectangularly shaped:

$$x(t) = \begin{cases} \cos\left(2\pi f_0 t + \mu t^2\right), |t| \le \tau/2 \\ 0, |t| > \tau/2 \end{cases}, \tag{7}$$

where: f_0 - carrier frequency, $\mu = \Delta\omega/2\tau$ - rate of frequency sweep, $\Delta\omega$ - swept spectrum bandwidth, τ - expanded pulse width.

The average sampled frequency is chosen, thus the analyzed frequency band is equal to the standard voice band channel (fs = 3.1kHz). The time samples are set to N = 127 and are calculated according to the equation:

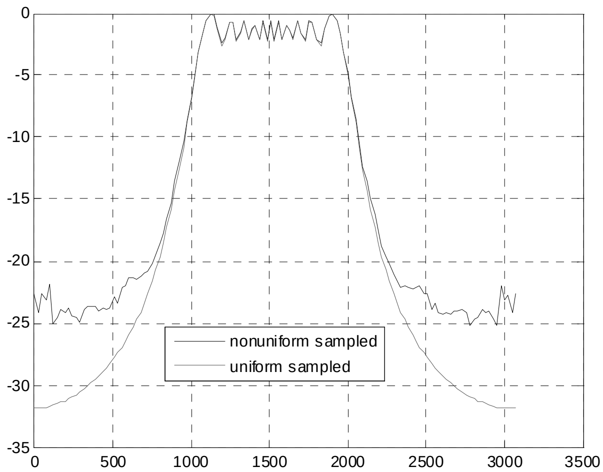
$$t_k = kT + \delta_k, 0 \le k \le N-1, \quad (8)$$

where $T = 1/fs$ - uniform sampling time, δ_k - deviation from the uniform sampling law.

The uniform sampling law deviation satisfies the equation $|\delta_k| \le {}^1\!/_2 T$ and is normally distributed at the selected boundary. The routine selects the optimal time instants among 10000 independent simulations, where the optimal sampling times are distinguished with the smallest condition number of the basic matrix [17].

The frequency estimation method follows the above described procedures. The first step is recognized as digital frequency shifting of the nonuniformly sampled data to translate the signal frequency band to the baseband. The second step accomplishes the frequency estimation method according to the Eq. (6).

The simulation conditions and frequency estimation results are represented in Fig. 3. The signal frequency band before digital frequency shifting is set to 186 ÷ 189.1kHz, which is down-converted to 0 ÷ 3.1kHz.



Conditions: $f_0 = 60fs + 1\text{kHz}$, $\mu = 8.10^4 \text{rad/s}^2$
(τ = 41ms, Δf = 1.044kHz)
fs = 3.1kHz (T = 0.3226ms)

Fig.3. Estimated signal spectrum.

Therefore, the high frequency signal is sampled with a 3.1kHz sampling frequency and its spectrum is shown below. The figure shown indicates that the proposed method correctly estimates the signal frequency response by using digital frequency shifting of the nonuniformly sampled data to the analyzed frequency band (baseband). At the same time the nonuniform sampling scheme is distinguished with a slightly increased sidelobe level in comparison with uniform sampling which cannot analyze data above the Nyquist limit.

5. CONCLUSION

The simulation results show that the proposed method successfully estimates signal the spectrum response even when. The method allows using low cost analog-to-digital converters (ADC) to analyze the high frequency signals when the nonuniformly sampling scheme is used. The excellent computation effectiveness allows implementing the digital signal processing procedure in the microcontroller core, which minimizes the device cost and increases system performance.

REFERENCES

1. Yuou P., Baert E., Loutre M. F.: *Spectral analysis of climate data*, Surv. Geophys., vol. 17, pp. 619 - 663, 1996.
2. Tropea C.: *Laser doppler anemometry: Recent developments and future challenges*, Meas. Sci. Technol., vol. 6, pp. 605 - 619, 1995.
3. Rolain Y., Schoukens J., Vandersteen G.: *Signal reconstruction for non-equidistant finite length sample sets: A "KIS" approach*, IEEE Trans. Instrum. Meas., vol. 47, pp. 1046 - 1052, 1998.
4. Pribic R.: *Radar irregular sampling*, ICASSP 2004, vol. 3, pp.933 - 936, 2004.
5. Koh J., Sarkar T., Wicks M.: *Spectral analysis of nonuniformly sampled data using a least square method for application in multiple PRF system*, Proceedings of IEEE International Conference on Phased Array Systems and Technology 2000, pp. 141 - 144.
6. Lomb N.: *Least square frequency analysis unequally sampled data*, Aerophysics and space science, vol. 39, pp. 447 - 462, May, 1975.
7. Tuszynski M., WojtkiewiczA.: *Application of the Dirichlet transform in analysis of nonuniformly sampled signals*, 1992 IEEE International Conference on Acoustics, Speech, and Signal Processing ICASSP-92, May 23-26, 1992, San Francisco, USA, vol. 5, pp. 25 - 28.
8. Chen W., Zhou G., Giannakis G.: *Velocity and acceleration estimation of doppler weather radar/lidar signals in coloured noise*, IEEE International Radar Conference, 1995, pp. 2052 - 2055.
9. Liu Q. H., Nguyen N.: *An accurate algorithm for nonuniform fast Fourier transform (NUFFT's)*, IEEE Microwave and guided wave letters, vol. 8, no. 1, 1998, pp. 18 - 20.
10. Bland D., Laakso T., Tarczynski A.: *Analysis of algorithms for nonuniform-time discrete Fourier transform*, IEEE International Symposium on Circuits and Systems ISCAS '96, vol. 2 , 12-15 May 1996, pp. 453 - 456.
11. Bland D., Laakso T., Tarczynski A.: *Spectrum estimation of non-uniformly sampled signals*, Proceedings of the IEEE International Symposium on Industrial Electronics, ISIE '96, vol. 1, 17-20 Jun 1996, pp. 196-200.
12. Ivanov R.: *One dimensional signal processing*, Gabrovo Print Ltd, 1989.
13. Openhaim A. V., Willsky A. S., Young I. T.: *Signals and systems*, Prentice – Hall, Inc., 1983.
14. Alavi H., Abbasi H.: *Low aliasing frequency estimation using nonuniformly sampled data*, Proceedings of ICCS'94, Singapore, pp. 205 - 209.
15. Luther U., Rost K.: *Matrix exponentials and inversion of confluent Vandermonde Matrices*, Electronic Transactions in Numerical Analysis, vol. 18, 2004, pp. 91 - 100.
16. Parker F.: *Inverses of Vandermonde matrices*, Amer. Math. Monthly, vol. 71, 1964, pp. 410 - 411.
17. Miletiev R.: *Methods and algorithms for automated real - time signal processing of nonuniformly sampled data*, PhD thesis, Technical University of Sofia, 2005.

ANALIZA SYGNAŁÓW PRÓBKOWANYCH NIEREGULARNIE POWYŻEJ WARTOŚCI GRANICZNEJ NYQUIST'A

Streszczenie

Proponuje się nową metodę analizy widmowej danych próbkowanych nieregularnie powyżej granicznej wartości Nyquist'a, opartej na cyfrowym przesunięciu widmowym do pasma podstawowego, filtracji optycznej i następnie analizie widmowej poniżej wartości granicznej Nyquist'a. Metoda odznacza się niską oceną częstotliwości aliasowania i wystarczającą skutecznością obliczeń.

Podane są podstawy matematyczne metody; przedstawione zosdtały też wyniki symulacji dla zilustrowania podstaw metody.